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# A PEER-TO-PEER REFUELING STRATEGY USING LOW-THRUST PROPULSION

**Atri Dutta[*] and Nitin Arora [†] and Ryan P. Russell [‡]**

The problem of minimum-fuel, time-fixed, low-thrust rendezvous is addressed with the particular aim of developing a solver to determine optimal low-thrust Peer-to-Peer (P2P) maneuvers, which will be an integral part of distributed low-thrust servicing missions for multiple satellites. We develop the solver based on an indirect optimization technique and utilize the well-known shooting method to solve the two-point boundary value problems associated with the forward and return trips of a P2P maneuver. We finally demonstrate the application of the tool in determination of the optimal P2P maneuvers required for a low-thrust P2P mission for multiple satellites moving in a circular orbit. The development of this solver is a first step in the direction of studying low-thrust servicing missions for multiple satellites in circular constellations.

## INTRODUCTION

Lawden's primer vector theory[1] has been successfully used for the study of optimal orbital transfers, for the case of high thrust and low-thrust maneuvers. Typically, the maneuvers are impulsive or high thrust when a spacecraft employs a chemical propulsion system; while they are low-thrust when the spacecraft employs an electric, solar-electric, or ionic propulsion system. Both minimum time and minimum fuel problems have been extensively studied for the case of low-thrust orbital transfers.[2,3,4] The determination of optimal low-thrust maneuvers involves formulation of an optimal control problem. The necessary conditions for optimality provide a set of differential equations, which along with the boundary conditions, lead to a two-point boundary value problem (TPBVP). A wide range of methods, based on direct and indirect optimization, exist in the literature for solving the low-thrust TPBVP.[4,5,6,7,8,9] In this paper, we develop a solver for the minimum fuel low-thrust rendezvous problem, particularly with the aim of its application to the problem of on-orbit servicing missions.

On-orbit servicing (OOS) provides immense benefits by extending the lifetime of space assets through replenishment and repairs, and by improving their performance through upgrades.[10,11] A number of studies have been made on the technological and economic feasibility of OOS missions.[12,13,14,15,16] All future servicing operations for a system of multiple spacecraft (satellite constellation, formation flying spacecraft, fractionated spacecraft, etc.) would require several satellites to be serviced as part of a single mission. Key technologies for such missions, such as automated rendezvous and capture, fuel exchange, etc. have been demonstrated during the successful tests carried out by the Orbital Express program of DARPA.[17] The problem of servicing multiple satellites

[*]Ph.D., D. Guggenheim School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0150. Email: a3d@gatech.edu.
[†]Graduate Student, D. Guggenheim School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0150. Tel: , Email: n.arora@gatech.edu.
[‡]Assistant Professor, D. Guggenheim School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0150. Email: ryan.russell@gatech.edu.

using a single service vehicle,[18,19] or using a distributed method incorporating Peer-to-Peer (P2P) maneuvers has been studied extensively for the case of satellites in circular constellations, and when the satellites employ chemical propulsion systems.[20,21] Therefore, the maneuvers are impulsive in nature in all of these studies. Furthermore, these studies considered the case of refueling because among all servicing operations, refueling is perceived to be a low-risk operation because it is performed at the end of lifetime of the satellites, and also refueling capabilities in satellites requires minimal design changes.[11,22] The benefits of refueling is not limited to lifetime extension of satellites; the provision of refueling capabilities imparts enhanced flexibility by allowing for maneuvers that would have significantly decreased the lifetime of satellites owing to high fuel consumption. Also, the provision of refueling capabilities enable new missions such as space-based lasers, or a system of satellites in low altitude high drag orbits.[23,24,22]

The underlying idea of a distributed refueling mission for multiple spacecraft is as follows: Suppose, there are 10 satellites to be refueled. An external spacecraft can refuel 5 of these satellites, which in turn redistribute the fuel among the remaining satellites by engaging in Peer-to-Peer maneuvers. During a P2P maneuver, two satellites engage in a rendezvous, exchange fuel, and then the maneuvering (active) satellite returns back to its original orbital position. References 21, 25 demonstrate the utility of P2P maneuvers in reducing the fuel expenditure during servicing missions. However, these studies considered that the satellites employ chemical propulsion systems, so that the maneuvers are impulsive in nature. This paper is a step towards achieving a distributed low-thrust refueling mission for multiple satellites. In such a mission, the objective is to incur minimum fuel expenditure during all ensuing maneuvers, and simultaneously ensure that all satellites maintain a minimum required amount of fuel at the end of the process. In order to determine the optimal set of maneuvers, we need to first obtain all possible P2P maneuvers and the fuel expenditure associated with each of them. The determination of fuel expenditure associated with all P2P maneuvers possible in a constellation requires the solution of numerous rendezvous problems. For instance, a P2P refueling problem for a constellation with $10$ satellites requires the solution of around $100$ rendezvous problems, each of which requires the (global) solution of a non-linear programming (NLP) problem. Herein lies the main challenge of the study of low-thrust P2P missions, because solving even a single NLP is computationally intensive, and a certificate of global optimality is non-existent. Therefore, there can be issues with the convergence to a local minimum rather than the global minimum.

We address these issues in two ways. Firstly, for each rendezvous problem, we find several local optimum, and choose the best of all of them. The idea is to obtain a good solution, that is, the obtained solution is either the global optimum, or very close to it. Secondly, we recognize that the servicing problem is a discrete optimization problem and the optimal answer to the servicing problem (that is, satellite pairs that undergo fuel exchange) will not change even if the cost associated with the decision variables is slightly inexact. In other words, we can arrive at the optimal matching of satellites if the developed NLP solver provides a good quality sub-optimal solution (ensured by consideration of several local minimum) for each NLP. Hence, the development of an efficient solver for P2P maneuvers is crucial to the study of low-thrust servicing missions. In this paper, we develop such a solver to determine the optimal transfers required for both the forward and return trips of a P2P maneuver. Given an initial guess from a bounded random feed, the equations of motion and the adjoint equations are integrated using a ODE87 integrator[26] and the well-known shooting method is then used to predict a new set of initial guesses for the next iteration until the convergence criteria is met. In the implementation of the shooting method, we use the derivatives calculated analytically from the propagation of the state transition matrix.[4] We also use the adjoint control transformation technique[27] in order to obtain good initial guesses for solving the TPBVP. In the forthcoming sections, we describe in detail our methodology, and finally demonstrate the application of our solver to a P2P refueling problem.

## P2P REFUELING STRATEGY

In this section, we will discuss in detail the P2P refueling strategy, and point out the significance of the developed solver in its determination of the optimal P2P maneuvers for multiple satellites in a circular constellation.

### Notations

Let us consider a circular constellation consisting of $n$ satellites, distributed evenly over $n$ orbital slots in a circular orbit of radius $R$. Let the set of $n$ satellites be given by $\mathcal{S} = \{s_i : i = 1, 2, \ldots, n\}$. Let the set of $n$ orbital slots be given by $\Phi = \{\phi_i \in [0, 2\pi) : i = 1, 2, \ldots, n,\ \phi_i \neq \phi_j\}$. Also, let the fuel content of satellite $s_i$ at time $t$ be denoted by $f_{i,t}$. In particular, let the initial fuel content of satellite $s_i$ be denoted by $f_i^-$ and the final fuel content be denoted by $f_i^+$; that is, $f_i^- = f_{i,0}$ and $f_i^+ = f_{i,T}$, where $T$ is the time allotted for refueling. Also, let $\underline{f}_i$ denote the minimum amount of fuel for the satellite $s_i$ to remain operational. Finally, let $\bar{f}_i$ denote the maximum fuel capacity of the same satellite. *Fuel-sufficient* satellites are those which have at least the requisite amount of fuel; the remaining satellites are *fuel-deficient*. The fuel-sufficient satellites have excess fuel and are thereby capable of sharing this fuel with other satellites in the constellation. The fuel-deficient satellites are depleted of fuel. Let $\mathcal{I}_{s,0}$ denote the set comprised of indices of the fuel-sufficient satellites, and let $\mathcal{I}_{d,0}$ denote the set having as elements the indices of the fuel-deficient ones. Clearly, $\mathcal{I}_{s,t} = \{i : f_{i,t} \geq \underline{f}_i\}$, $\mathcal{I}_{d,t} = \{i : f_{i,t} < \underline{f}_i\}$ and $\mathcal{I}_{s,0} \cup \mathcal{I}_{d,0} = \mathcal{I}$, where $\mathcal{I} = \{1, 2, \ldots, n\}$.

The objective of P2P refueling is therefore to achieve $f_i^+ \geq \underline{f}_i$ for all $i \in \{1, 2, \ldots, n\}$ by expending the minimum amount of fuel during the ensuing orbital transfers. During a P2P refueling transaction between a fuel-sufficient and a fuel-deficient satellite, one of them (henceforth referred to as the *active* satellite) performs an orbital transfer to rendezvous with the other satellite (henceforth referred to as the *passive* satellite). In general, after a fuel exchange takes place between a fuel-sufficient satellite, the active satellite can return to any available orbital slot left vacant by another active satellite. In other words, the active satellites may be allowed to interchange their orbital positions during their return trips. However, the discrete optimization problem associated with this problem is NP-hard.[28] Hence, for the sake of simplicity, we consider that each active satellite returns to its original orbital slot. Let us consider a simple constellation with $n = 4$ four satellites $s_1$, $s_2$, $s_3$, and $s_4$, occupying the orbital slots $\phi_1$, $\phi_2$, $\phi_3$ and $\phi_4$ respectively. Of these, suppose $s_1$ and $s_4$ are fuel-sufficient, while $s_2$ and $s_3$ are fuel-deficient. Figure 1(a) shows this constellation and two P2P maneuvers. In one P2P maneuver, satellite $s_1$ transfers and rendezvous with $s_2$, delivers fuel to the latter, and finally returns back to its original position $\phi_1$. The satellite $s_2$ remains in its orbital slot through out the process. Similarly, in the other P2P maneuver, satellite $s_3$ transfers and rendezvous with $s_4$, receives fuel from the latter, and finally returns to its original position $\phi_3$. Note that the satellite $s_3$ being fuel-deficient does not necessarily mean that it is depleted of fuel. In other words, a fuel-deficient satellite ($s_3$ in the example above) can be the active satellite in a P2P maneuver.

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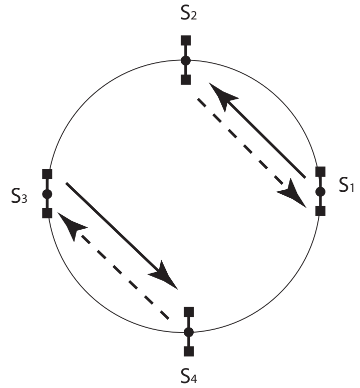
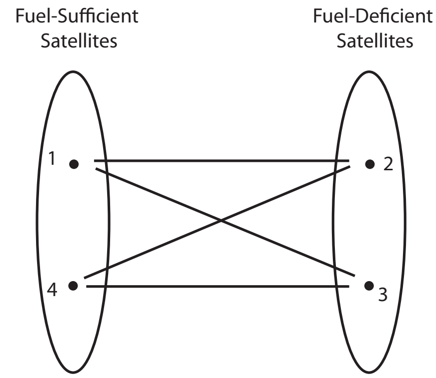



(a) P2P Maneuvers




(b) Bipartite Graph

**Figure 1. Example of Peer-to-Peer Refueling Problem.**

For convenience, let $\mathcal{J}_{s,t} = \{j : \sigma_t(\phi_j) = s_i, i \in \mathcal{I}_{s,t}\}$ denote the index set of orbital slots occupied by fuel-sufficient satellites at time $t$, and let $\mathcal{J}_{d,t} = \{j : \sigma_t(\phi_j) = s_i, i \in \mathcal{I}_{d,t}\}$ denote the index set of orbital slots occupied by fuel-deficient satellites at time $t$. For instance, for the example depicted in Figure 1(a), we have $\mathcal{J}_{s,0} = \{1, 2\}$, and $\mathcal{J}_{d,0} = \{3, 4\}$. Furthermore, for each satellite $s_i$, we denote the mass of its permanent structure by $m_{\text{spi}}$ and the specific thrust of its engine by $I_{\text{spi}}$. We denote the gravitational acceleration on the surface of the earth by $g_0$. For each satellite $s_i$, we therefore define the characteristic constant as $c_{0i} = g_0 I_{\text{spi}}$. Furthermore, let us denote by $p_{ij}^k$ the amount of fuel expended by satellite $s_k$ to move from the orbital slot $\phi_i$ to $\phi_j$.

## P2P Formulation

The P2P refueling problem can be efficiently formulated using constellation graphs,[29,30] in which each node represents an orbital slot and an edge represents a P2P maneuver between the slots. We follow the same approach in this paper. To this end, let us consider an undirected bipartite graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with the two partitions being $\mathcal{J}_{s,0}$ and $\mathcal{J}_{d,0}$. There exists an edge $\langle i, j\rangle \in \mathcal{J}_{s,0} \times \mathcal{J}_{d,0}$ if the satellites $s_i$ and $s_j$ can engage in a P2P refueling transaction such that at the end of the refueling process, both the satellites end up being fuel-sufficient. Let $\mathcal{E} \subseteq \mathcal{J}_{s,0} \cup \mathcal{J}_{d,0}$ be the set of all edges in $\mathcal{G}$. To each edge $\langle i, j\rangle \in \mathcal{E}$, we assign a cost $c_{ij}$ that equals the fuel expenditure incurred during a P2P refueling transaction between the two satellites. For instance, for the simple constellation depicted in Figure 1(a), there can be four P2P maneuvers possible and these are depicted as four edges in the graph in Figure 1(b). Recognizing that either of the two satellites engaged in a P2P refueling transaction can be the active one, we define the cost associated with each edge $\langle i, j\rangle$ of the graph as follows:

$$c_{ij} = \begin{cases} p_{ij}^i + p_{ji}^i, & \text{if } s_i \text{ can be active, but } s_j \text{ cannot,} \\ p_{ji}^j + p_{ij}^j, & \text{if } s_j \text{ can be active, but } s_i \text{ cannot,} \\ \min\{p_{ij}^i + p_{ji}^i, p_{ji}^j + p_{ij}^j\}, & \text{if either } s_i \text{ or } s_j \text{ can be active,} \\ \infty, & \text{if neither } s_i \text{ nor } s_i \text{ can be active.} \end{cases} \tag{1}$$

We are interested in a P2P solution (that is, a set of P2P maneuvers) that would incur minimum cost during the refueling process. For instance, the set of feasible P2P solutions for the simple constellation depicted in Figure 1(a), there are two possible P2P solutions are $\{(1, 2), (3, 4)\}$, and

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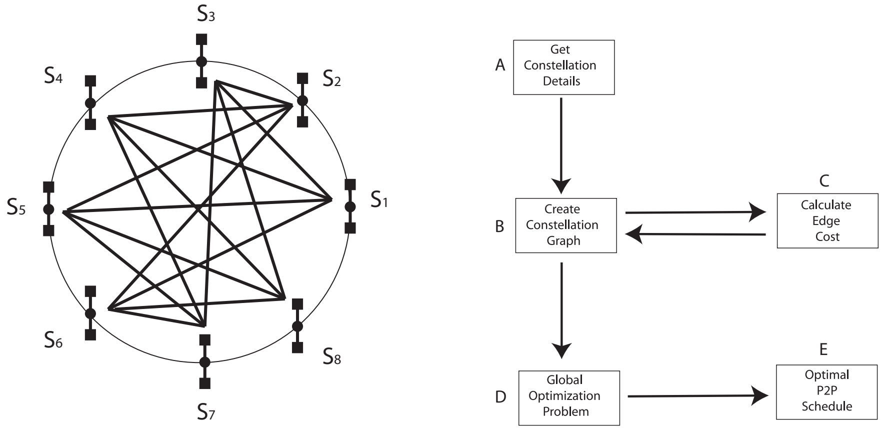


(a) Possible P2P Maneuvers (b) P2P Algorithm

**Figure 2. P2P Optimization Problem.**

$\{(1,3),(2,4)\}$. Of course, for this example, this is simple. However, with increasing number of satellites in the constellation, the number of edges (possible P2P maneuvers) grows fast. For instance, for a constellation of $n = 8$ satellites, there are (potentially) $4 \times 4 = 16$ P2P maneuvers (see Figure 2(a)), out of which we need to choose $4$ (this can be done in $16C4 = 1820$ ways). Hence, we need an efficient formulation in order to solve these problems. To this end, we associate a binary decision variable with each P2P maneuver to denote whether that P2P maneuver is selected in the P2P solution. Note that since all fuel-deficient satellites need to be refueled, the number of P2P maneuvers that are required to achieve fuel-sufficiency in the constellation must equal the number of fuel-deficient satellites in the constellation. Hence, we are interested in a set $\mathcal{M} \subseteq \mathcal{E}$ of $|\mathcal{I}_{d,0}|$ edges that has minimum total cost and such that all fuel-deficient satellites are involved in fuel transactions. Let us also associate with each edge $\langle i,j \rangle \in \mathcal{E}$ a binary variable $x_{ij}$ defined as

$$x_{ij} = \begin{cases} 1, & \text{if } \langle i,j \rangle \in \mathcal{M}, \\ 0, & \text{otherwise}. \end{cases} \tag{2}$$

We have the following optimization problem that yields the optimal P2P solution (for details, see Ref. 30:

$$\min_{\mathcal{M} \subseteq \mathcal{E}} \sum_{\langle i,j \rangle \in \mathcal{E}} c_{ij} x_{ij}, \tag{3}$$

such that

$$\sum_{j \in \mathcal{J}_{d,0}} x_{ij} \leq 1, \text{ for all } i \in \mathcal{J}_{s,0}, \tag{4}$$

$$\sum_{i \in \mathcal{J}_{s,0}} x_{ij} = 1, \text{ for all } j \in \mathcal{J}_{d,0}, \tag{5}$$

Constraint (4) implies that a fuel-sufficient satellite can be assigned to at most one refueling transaction, while constraint (5) implies that a fuel-deficient satellite has to be assigned to a refueling transaction.

The algorithm for the determination of the optimal P2P maneuvers is summarized in Fig. 2(b). Our particular interest will be the block 'C' that calculates the cost of the P2P maneuvers. The cost can be calculated by solving two low-thrust transfer problems, one associated with the forward trip, and the other associated with the return trip. The determination of a locally optimal low-thrust transfer requires the solution of a non-linear programming (NLP) problem. Since (potentially) numerous low-thrust problems need to be solved ($16 \times 2 \times 2 = 64$ for constellation in Fig. 2(a), $16$ P2P maneuvers each with forward and return trips and choice of an active satellite) in order to set up the discrete optimization problem (block 'D' of Figure 2(b)), an efficient determination of each low-thrust maneuver is crucial for the solution of the overall P2P problem. In this paper, we follow the approach of Ref. 4 in order to develop an efficient tool to determine the fuel expenditures corresponding to the forward and the return legs of a P2P maneuver. The tool corresponds to the block 'C' of Figure 2(b). Note that in the case of a NLP, we do not have a certificate of global optimality for a solution, that is, the solution to a NLP would only be a local minimum. Hence, for each transfer problem, we find several local optima and consider the best of these. By following this strategy, we intend to find a solution that is either the global optimum or very close to it. In the forthcoming sections, we discuss in detail the forward and return legs of a P2P maneuver.

## P2P FORWARD TRIP

Let us consider a P2P maneuver between two satellites $s_i$ and $s_j$. Without loss of generality, we may assume that the satellite $s_i$ is active, that is, it performs a low-thrust transfer to rendezvous with $s_j$, undergoes a fuel exchange, and then performs another low-thrust transfer to return back to its original position. We assume that we are given a fixed time $t_f$ for each leg of the P2P maneuver. In this section, we discuss in detail the optimal control problem associated with the forward trip of satellite $s_i$. For the convenience of discussion, we drop the index $i$ from the transfer trajectory equations. Also, note that the fixed time $t_f$ represents an upper bound on the time of completion of the maneuver.

### Mathematical Formulation

Let the mass of the satellite $s_i$ at time t be given by $m(t)$. Since minimizing fuel expenditure during the transfer is equivalent to maximizing the final mass at the end of the transfer, the objective function can be written as

$$J = -k\, m_f \tag{6}$$

where $m_f = m(t_f)$. The negative sign takes care of the fact that we pose the problem as a minimization problem. The equations of motion and the rendezvous requirements provide respectively the dynamic and terminal constraints for the optimal control problem. Let $\mathbf{r}(t)$ and $\mathbf{v}(t)$ denote the radius vector and the velocity vector of the maneuvering vehicle respectively at time $t$. Also, let the thrust be given by $T\mathbf{u}$, where $T$ is the magnitude of the thrust vector and $\mathbf{u}$ is a unit vector in the direction of the thrust. Therefore, $\mathbf{u}^T\mathbf{u} = 1$. Now, the dynamics can be written as

$$\dot{\mathbf{r}} = \mathbf{v} \tag{7}$$

$$\dot{\mathbf{v}} = \mathbf{g}(\mathbf{r}) + \frac{T}{m}\mathbf{u} \tag{8}$$

$$\dot{m} = -\frac{T}{c} \tag{9}$$

where $\mathbf{g}(\mathbf{r})$ is the position dependent acceleration term (in our case the two-body gravity term), $r = |\mathbf{r}|$, $c = g_0 I_{\mathrm{sp}}$, $g_0$ is the acceleration due to gravity at the surface of Earth and $I_{\mathrm{sp}}$ is the specific impulse of the engine. The states of the system are given by $\mathbf{x} = [\mathbf{r}\,\mathbf{v}\,m]^T$. For a rendezvous problem, we have the following initial and final constraints.

$$\mathbf{r}(0) = \mathbf{r}_0^*, \quad \mathbf{v}(0) = \mathbf{v}_0^*, \quad \mathbf{r}(t_f) = \mathbf{r}_f^*, \quad \mathbf{v}(t_f) = \mathbf{v}_f^* \tag{10}$$

where superscript '$*$' is used to denote a reference value (known a priori). Also, let us denote the initial mass of the maneuvering vehicle by $m_0$, that is, $m_0 = m(0)$. The Hamiltonian for the system can then be written as

$$H = \mathbf{\Lambda}_r^T \mathbf{v} + \mathbf{\Lambda}_v \left( \mathbf{g}(\mathbf{r})\,\mathbf{r} + \frac{T}{m}\mathbf{u} \right) + \Lambda_m \left( -\frac{T}{c} \right) + \eta \left( \mathbf{u}^T \mathbf{u} - 1 \right) \tag{11}$$

The first order necessary conditions for optimality can then be written as

$$\dot{\mathbf{\Lambda}} = - \left( \frac{\partial H}{\partial x} \right)^T, \tag{12}$$

$$0 = \left( \frac{\partial H}{\partial \mathbf{u}} \right)^T = \mathbf{\Lambda}_v \frac{T}{m} + 2\eta\mathbf{u}, \tag{13}$$

and

$$\Lambda_{mf} = -k. \tag{14}$$

Equation (13) implies that the thrust vector has to be applied in the direction opposite to that of the primer vector $\mathbf{\Lambda}_v$, that is, the costates associated with the velocity vector of the maneuvering vehicle. Eliminating $\mathbf{u}$ from the Hamiltonian, we have

$$H = \mathbf{\Lambda}_r^T \mathbf{v} + \mathbf{\Lambda}_v^T \left( \mathbf{g}(\mathbf{r}) \right) + \frac{T}{m}\$ \tag{15}$$

where $\$$ is a switching function defined by

$$\$ = \Lambda_v + \frac{\Lambda_m m}{c} \tag{16}$$

Clearly, the Hamiltonian is minimized for the following control action (Pontryagin's Minimum Principle[31]):

$$T = \begin{cases} T_{\max}, & \text{if } \$ > 0, \\ 0, & \text{otherwise.} \end{cases} \tag{17}$$

The control law illustrated in the above example is the classical example of a bang-bang control law[31] in which the value of the switching function dictates the thrusting structure. For $\$ = 0$, we have a singular arc, and hence we would require a singular control law. However, as singular arcs are rarely encountered in realistic spacecraft trajectory problems, we ignore the case $\$ = 0$ for the present analysis.

Equation (12) can be written as

$$\dot{\mathbf{\Lambda}}_r = -G^T \mathbf{\Lambda}_v \tag{18}$$

$$\dot{\mathbf{\Lambda}}_v = -\mathbf{\Lambda}_r \tag{19}$$

$$\dot{\Lambda}_m = -\Lambda_v \frac{T}{m^2} \tag{20}$$

where $G = \partial\mathbf{g}(\mathbf{r})/\partial(\mathbf{r})$ is the two-body gravity-gradient matrix. It should be noted that as the problem is formulated as a fixed-time transfer, accordingly there are no associated transversality conditions that need to be satisfied[31] on the flight time and the Hamiltonian values. However, the Hamiltonian is an integral of the system and is useful for checks on numerical consistency.

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## Solution Technique

The determination of the locally optimal low-thrust trajectory requires the solution of a two-point boundary value problem (TPBVP) given by equations $(2)$-$(4)$, $(12)-(14)$, and the boundary conditions given in $(5)$. The set of unknowns for the TPBVP is given by

$$\mathbf{U} = [\mathbf{\Lambda}_r(0) \quad \mathbf{\Lambda}_v(0)]^T, \tag{21}$$

while the set of terminal constraints required to be satisfied are given by

$$\mathbf{C} = [\mathbf{r}(t_f) - \mathbf{r}_f^* \quad \mathbf{v}(t_f) - \mathbf{v}_f^*]^T. \tag{22}$$

Note that $\Lambda_{m0}$ is also an unknown in the problem. However, we eliminate this unknown by taking advantage of the condition $\Lambda_{mf} = -k$, and the fact that $\Lambda_m$ is a decreasing function of time. We simply start with a value of $\Lambda_{m0} = -1$ so that $\Lambda_{mf}$ is negative and equals $-k$.[4] We use the well-known shooting method in order to solve the TPBVP associated with the forward trip. An important step in the implementation of the shooting method is the calculation of $\partial C/\partial U$ in each iteration of the method. As we will illustrate, the derivatives can be calculated efficiently by propagating the state transition matrix along with the equations of motion in each iteration. A modified, fast ODE87 integration algorithm[26] is used to perform the integrations in each iteration of the shooting method. At switching time when $\$$ crosses the zero boundary, the variable step integrator uses a back stepping algorithm to accurately stop at the exact stopping time, hence avoiding any discontinuities in the derivatives during successive iterations. Note that discontinuities will exist if successive iterations lead to different number of thrust arcs. Convergence is still possible unless chatter between structures is induced. The implicit bang bang structure eliminates the need to provide an explicit thrusting structure a priori.[4,32]

## Using Analytical Derivatives

As mentioned above, accurate partial derivatives of the final constraint with respect to initial unknowns must be provided for successful implementation of the shooting method. The accurate calculation of the derivatives is very important, given the highly non-linear nature of the problem under investigation. Even though the implementation of analytical derivatives can be tedious, the speed and accuracy outweighs this drawback. The procedure for obtaining analytical derivatives requires the use of state transition matrix along the continuous thrust or no-thrust portions along with a transition function that accounts for perturbations in switching times in successive iterations. A chain rule finally stacks all of the transition functions to obtain the required sensitivities of the final states with respect to the initial states (Refs. 4, 32). The state transition matrix over the continuous trajectory arcs is obtained by integrating the following equation along with the state equations at each integrator call.

$$\Phi(\dot{t}, t_0) = \frac{\partial f}{\partial Y}_t \Phi(t, t_0) \tag{23}$$

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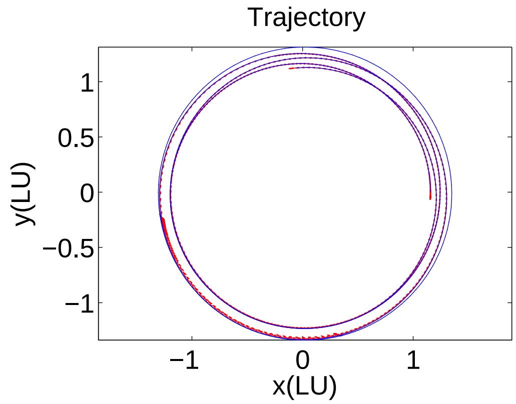


**Figure 3. Trajectory of the active satellite (Example 1).**

where initially $\Phi(t_0, t_0)$ is a $14 \times 14$ identity matrix. Below is the expression for $\frac{\partial f}{\partial Y}$ which is a $14 \times 14$ matrix,

$$\begin{pmatrix} 0_3 & I_3 & 0_3 & 0_3 & 0_3 & 0_3 \\ G & 0_3 & (\frac{\mathbf{\Lambda}_v}{\Lambda_v})(\frac{T}{m^2}) & 0_3 & -\frac{T}{m}(I_3/\mathbf{\Lambda}_v - \mathbf{\Lambda}_v\mathbf{\Lambda}_v{}^T/(\Lambda_v)^3) & 0_3 \\ 0_3 & 0_3 & 0 & 0_3 & 0_3 & 0 \\ -\frac{\partial(G^T\mathbf{\Lambda}_v)}{\partial\mathbf{r}} & 0_3 & 0_3 & 0_3 & -G^T & 0_3 \\ 0_3 & 0_3 & 0_3 & -I_3 & 0_3 & 0_3 \\ 0_3 & 0_3 & 2\Lambda_v\frac{T}{m^3} & 0_3 & -\mathbf{\Lambda}_v^T(\frac{T}{\Lambda_v m^2}) & 0 \end{pmatrix} \quad (24)$$

As already stated, STM maps partial derivatives of the state vector, at a given time with respect to an initial time , for a given continuous trajectory. But depending upon the problem being solved we may have multiple switchings at various times during the course of numerical integration. Hence the state transition matrix has to be joined together with a simple chain rule and transition function at these switches to accurately map the derivatives across these discontinuities.

Let there be N switches along the trajectory. The following expression summarizes the evaluation of the $\frac{\partial Y(tf)}{\partial Y(t0)}$

$$\Phi(t_f, t_{n+})\Psi_n\Phi(t_{n-}, t_{(n-1)+})......\Psi_1\Phi(t_{1-}, t_0) \quad (25)$$

where $\Psi_n$ is the partial derivative across the switching time, i.e partial derivative of state at $t_{n+}$ with respect to state at $t_{n-}$, given by following expression. For details, see References 4 and 32.

$$\mathbf{\Psi_n} = I_{14\times14} + (\dot{\mathbf{Y}}_{n+} - \dot{\mathbf{Y}}_{n-})((\frac{\partial S}{\partial \mathbf{Y}})/\dot{S})_{n-} \quad (26)$$

where $\dot{S}$ is given by $\dot{\Lambda}_v + \dot{\Lambda}_m m/c + \dot{m}\Lambda_m/c$ , and c is $g_0 Isp$.

The integrator does a function call at each successfully step to check whether a switching has occurred, and there is another function call if the switching has occurred which maps the derivatives across the switch. We emphasize that the switching time is calculated exactly via a bisection method. The gradient search algorithm involves using a variable gain which depends upon the norm of constraint at each step, this helps to speed the convergence of the TPBVP.

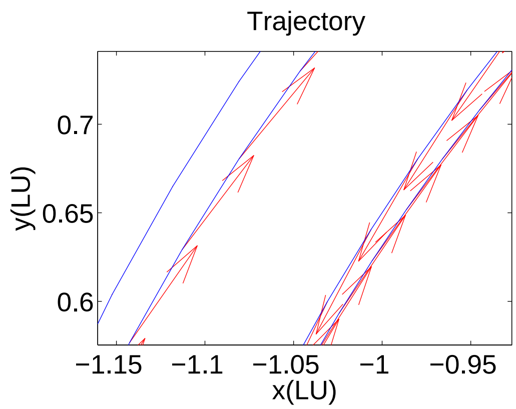


**Figure 4. Magnified Portion of Trajectory (Example 1).**

**Example 1** *A forward leg of a P2P.*

Let us consider a constellation of eight satellites, like the one shown in Fig. 1(a). Suppose, four of these satellites $s_1$, $s_6$, $s_7$, $s_8$ are fuel-sufficient, while the remaining four $s_2$, $s_3$, $s_4$, $s_5$ are fuel-deficient. The constellation is a circular orbit of altitude $1000$ Km. Let us focus on the forward trip of a P2P maneuver between $s_1$ and $s_3$, with $s_1$ being the active satellite. For solving the TPBVP associated with the forward trip, we use non-dimensionalized units such that $1$ LU is equal to the radius of the Earth, $\mu = 1$, and the time-period of the orbit is $2\pi$ TU. Suppose, the total time allotted for the P2P maneuver is $12$ orbital periods, with $6$ orbital periods for each leg of the maneuver, that is, $t_f = 12\pi$ TU. The optimal trajectory of $s_1$ is shown in Figure 3. Satellite $s_1$ makes $5$ revolutions in its transfer orbit, hence it makes an overall $5$ revs $+90$ degrees transfer. The thrusting portion of the transfer trajectory is marked in red in the figure, and the thrust directions are also marked. For clarity, a magnified view of a portion of the trajectory is provided in the Figure 4. The magnified view shows the thrust direction along the transfer trajectory. Figure 5 shows the variation of the three components of the costate associated with the position vector of the active satellite. Also, Figure 6 shows the variation of the three components of the costate associated with the velocity vector of the active satellite. Note that several (around 20) local minima are generated in order to obtain a good quality local solution (the deemed optimal). The result shown here is the best among all local solutions. The switching function is as seen in Figure 7. There are $4$ switches (thrust/no-thrust) along the trajectory with the control structure being T-C-T. The variation of the costate associated with the mass of the active satellite is given in Figure 8. As expected, $\Lambda_m$ is a decreasing function of time.

## P2P RETURN TRIP

In this section, we describe in details the optimal control problem associated with the return trip of the active satellite $s_i$. We also point out the differences compared to the forward trip optimal control problem.

### Mathematical Formulation

At the end of the forward trip, the satellites $s_i$ and $s_j$ undergo a fuel exchange, and after the fuel exchange, the active satellite $s_i$ returns to its original orbital slot $\phi_i$. The return trip starts

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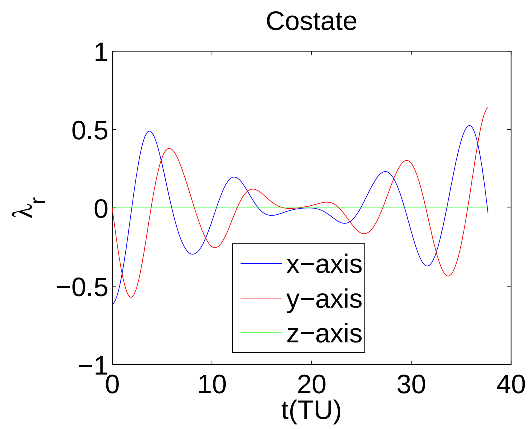


**Figure 5. Variation of $\Lambda_r$ (Example 1).**

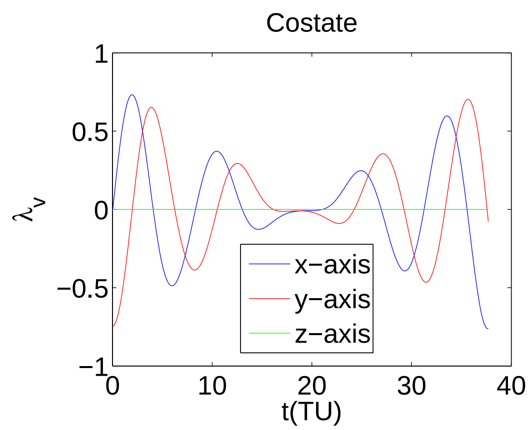


**Figure 6. Variation of $\Lambda_v$ (Example 1).**

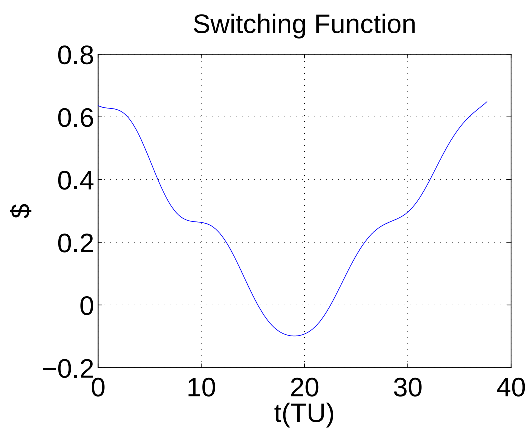


**Figure 7. Variation of Switching Function (Example 1).**

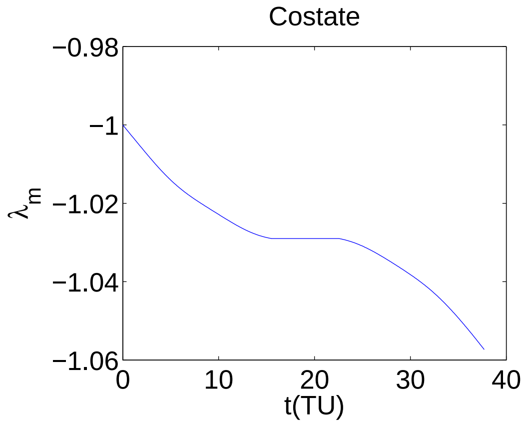


**Figure 8. Variation of $\Lambda_m$ (Example 1).**

at a time $t = t_f + t_e$, where $t_e$ is the time allowed for the fuel exchange, and ends at a time $t = t_f + t_e + t_r$, where $t_r$ denotes the time for the return trip. Let the amount of fuel exchanged between the satellites be denoted by $g_ij$. Depending on whether satellite $s_i$ is the fuel-sufficient or the fuel-deficient satellite, it loses or gains mass respectively as a result of the fuel exchange. In other words,

$$m_i(t_f + t_r) = \begin{cases} m_i(t_f) + g_{ij}, & \text{if } f_i^- < \underline{f}_i \\ m_i(t_f) - g_{ij}, & \text{if } f_i^- > \underline{f}_i \end{cases} \tag{27}$$

Equation(27) gives the initial mass for the return trip of the active satellite. Note that the amount of fuel exchanged affects the return trip optimal control problem, and thereby calls for a separate formulation.

**Table 1. Optimal Control Problems for P2P Maneuver.**

| States | Forward Trip | Return Trip ($) |
|---|---|---|
| $\mathbf{r}_0$ | Known | Known |
| $\mathbf{v}_0$ | Known | Known |
| $m_0$ | **Known** | **Unknown** |
| $\lambda_{\mathbf{r}0}$ | Unknown | Unknown |
| $\lambda_{\mathbf{v}0}$ | Unknown | Unknown |
| $\lambda_{\mathbf{m}0}$ | **Unknown** | **Known** |
| $\mathbf{r}_f$ | Known | Known |
| $\mathbf{v}_f$ | Known | Known |
| $m_f$ | **Unknown** | **Known** |
| $\lambda_{\mathbf{r}f}$ | Unknown | Unknown |
| $\lambda_{\mathbf{v}f}$ | Unknown | Unknown |
| $\lambda_{\mathbf{m}f}$ | **Known** | **Unknown** |

We want to minimize the amount of fuel expended during the return trip transfer, that is, equivalently, we minimize

$$J' = m_0 - m_f, \tag{28}$$

subject to the dynamics equations given in (7)-(10). In addition, there is a terminal constraint that needs to be satisfied because the active satellite has to return with at least a required amount of fuel at the end of the maneuver. This constraint can be written as:

$$m_f \geq \underline{m}, \tag{29}$$

where $\underline{m}$ is the required amount of mass that the satellite needs to have at the end of the maneuver. Introducing a slack variable $\alpha$,[31] where $\alpha \in \Re$, we can rewrite the constraint as

$$m_f - \underline{m} - \alpha^2 = 0. \tag{30}$$

For the optimal control problem, the augmented performance index can be written as

$$J'' = (m_0 - m_f) + \psi \left( m_f - \underline{m} - \alpha^2 \right) + \int_0^{t_f} \Lambda^T \left( f - \dot{x} \right), \tag{31}$$

where $\psi$ is the Lagrangian associated with the equality constraint (30). By the application of the calculus of variations and following a few simple mathematical steps, we can arrive at the necessary conditions of optimality. The necessary conditions include the (standard) Euler-Lagrange equations

$$\frac{\partial H}{\partial x} + \dot{\Lambda}^T = 0, \tag{32}$$

and

$$\frac{\partial H}{\partial u} = 0, \tag{33}$$

where $H$ is the Hamiltonian. The necessary conditions of optimality also yield the following:

$$\lambda_{m0} + 1 = 0, \tag{34}$$

$$\psi - \lambda_{mf} - 1 = 0, \tag{35}$$

and Let us consider the case when $\psi = 0$, that is, the inequality constraint of (29) is not active. This may happen when the active satellite returns with fuel more than the minimum required amount. In this case, we have $\lambda_{mf} = -1$ from equation (35), which is not possible because $\lambda_{m0} = -1$ and $\lambda_m$ is a monotonically decreasing function. Hence, it is not possible to have $\psi = 0$. In other words, the inequality constraint (29) has to be active for a solution to satisfy the necessary conditions. ($\alpha = 0$ if $\psi \neq 0$ from (37)). This means that we must have

$$m_f = \underline{m} \tag{36}$$

for the optimal solution. The return trip problem turns out to be a targeting problem, in which the final mass is known. The amount of fuel exchanged between the satellites is finally calculated as the difference of the initial mass or the return trip and the final mass of the forward trip.

**Solution Strategy**

Let us first note the difference between the two optimal control problems associated with the two legs of the P2P maneuver. All conditions remain the same except the boundary conditions involving mass and the costate associated with the mass. This difference is illustrated in Table 1. A shooting method similar to the forward trip is employed to solve the return trip.

$$\psi\alpha = 0. \tag{37}$$

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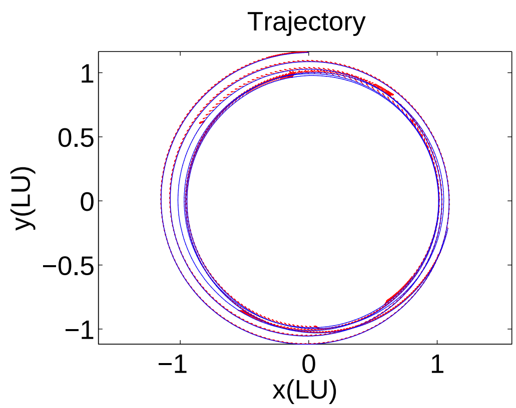


**Figure 9. Trajectory of the active satellite (Example 2).**

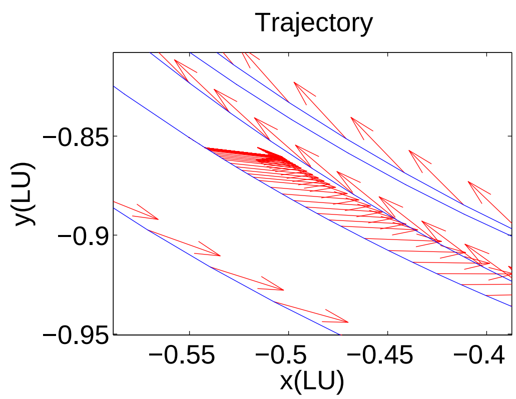


**Figure 10. Magnified Portion of Trajectory (Example 2).**

**Example 2** *Return leg of a P2P maneuver.*

Continuing on Example 1, let us particularly focus on the P2P maneuver between satellites $s_1$ and $s_3$. During the return trip, the satellite $s_1$ returns to its original orbital slot. Our solver converges to the trajectory depicted in Figure 9. A magnified portion of the trajectory is shown in Figure 10 in order to have an idea of the thrust direction on the trajectory. The transfer trajectory is comprised of five revolutions, hence it is a $5$ rev + $270$ deg. transfer. The variations of the costates and switching functions are given in Figures 12- 14. The thrusting structure here is T-C-T-C-T-C-T-C-T-C.

## P2P EXAMPLE

In this section, we provide additional example demonstrating the use of our developed solver for obtaining solutions for a full-scale low-thrust P2P refueling strategy. To this end, let us consider a circular constellation of $8$ satellites as shown in Figure 15(a). The altitude of the constellation is 1000 Km. Each satellite has a dry mass of 900 Kg. and a maximum wet mass of 1000 Kg. Each satellite employs low-thrust propulsion system with a specific impulse of 3000 sec. Satellites $s_1$, $s_6$, $s_7$, and $s_8$ are fuel-sufficient and are marked by a '+' sign. Let us consider that they have the

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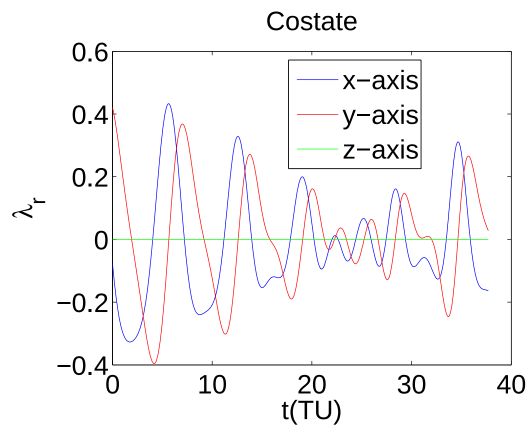



**Figure 11. Variation of $\Lambda_r$ (Example 2).**

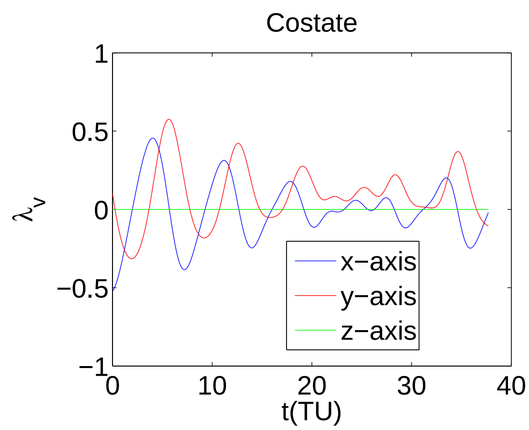



**Figure 12. Variation of $\Lambda_v$ (Example 2).**

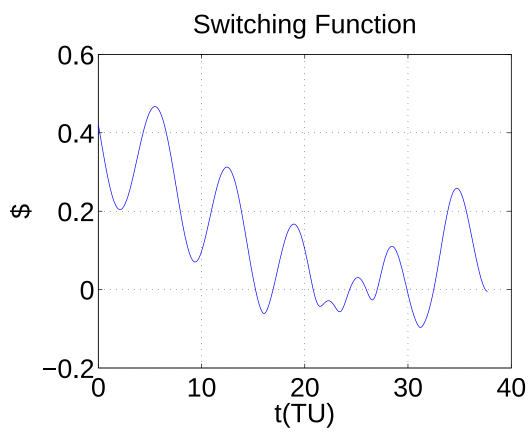



**Figure 13. Variation of Switching Function (Example 2).**

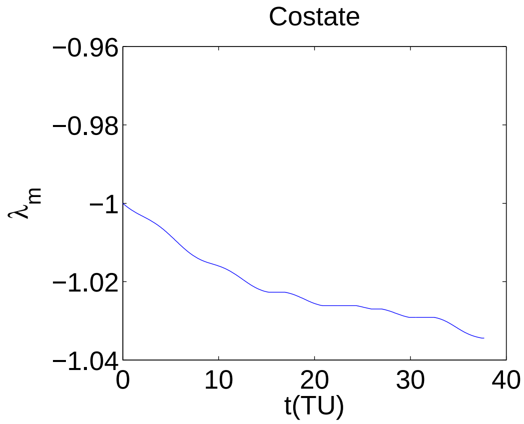
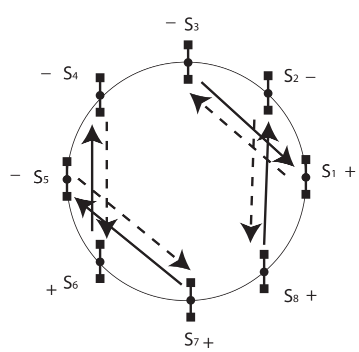
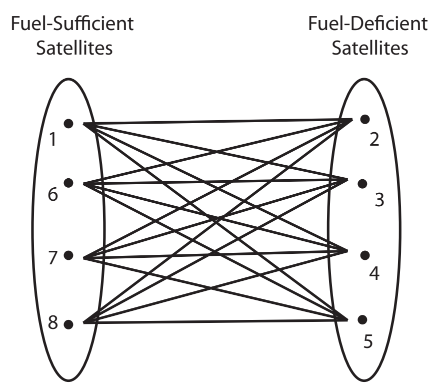



**Figure 14. Variation of $\Lambda_m$ (Example 2).**

maximum amount of fuel (possibly after they have been filled up completely by a refueling service vehicle launched from Earth). These satellites need to distribute the fuel to the remaining four satellites $s_2$, $s_3$, $s_4$, and $s_5$ which are fuel-deficient. Let us consider that they have a fuel amount of 1 Kg, 7 Kg, 2 Kg, and 1 Kg. respectively. Suppose, all satellites are required to maintain at least 35 Kg. of fuel on board. We allow a time of 6 orbital periods for each leg of all ensuing P2P maneuvers. Given these details, we construct a bipartite constellation graph, as shown in Figure 15(b). For each edge of the bipartite graph, our developed tool is invoked to yield the cost of the corresponding P2P maneuver. This is obtained by solving the forward and return trip TPBVP for two cases: (1) fuel-sufficient satellite is active, (2) fuel-deficient satellite is active. If any of these two cases is feasible, then the edge is allowed in the graph, otherwise the edge is considered to have infinite cost and simply dropped from the graph. Since $4$ fuel-sufficient satellites can pair up with any of the $4$ fuel-deficient satellites, we have to solve for $4 \times 4 \times 2 \times 2 = 64$ low-thrust transfer problems. On the




(a) P2P Maneuvers




(b) Bipartite Graph

**Figure 15. Example 3: Low-Thrust Peer-to-Peer Refueling Scenario.**

average, around 20 local optimum solutions are determined for each transfer. The best local solution is then selected as the optimal solution for that particular transfer. Approximately 200 differential corrector iterations are performed to converge to a local optimum. For integration, we use a variable step Dormand and Prince integrator[26] that is of 8$^{th}$ order and has 7$^{th}$ order error control. Also, the integrator is set to a unitless tolerance of 1E-14. Overall, the time taken to determine all transfers required to set up the discrete optimization problem for this constellation was approximately 75 min. using a computer having an Intel dual core processor (2.6Ghz) with 4 GB of RAM. Only single core was utilized for our computations.

Once all the P2P maneuvers are determined, we set up the bipartite constellation graph. Note that the bipartite graph is complete, that is, all possible P2P maneuvers are feasible in this case. In cases when the fuel-deficient satellite has very low amount of fuel (say, $s_2$, $s_4$, $s_5$), such that they cannot make the forward trip, their fuel-sufficient counterparts become the active satellites. Once this graph constructed, then we solve the global P2P optimization problem. In this case, we have a two-dimensional matching problem which can be solved in polynomial time.[33] (This is true only for the case when we restrict the active satellites to return to their original orbital slots; if we relax this constraint, the problem is NP-hard and we need an algorithm as in Ref. 34 in order to efficiently solve the discrete optimization problem). The optimal solution that is obtained for the example problem under consideration is shown in Figure 15(a). The forward trips of the active satellites are marked by solid arrows, while their return trips are marked by dotted arrows. Overall, around $8.2\%$ of total fuel in the constellation is expended during the refueling process. Note that three of the fuel-sufficient satellites are active because their corresponding fuel-deficient counterparts do not have sufficient fuel to make the forward trips. However, the satellite $s_3$ has enough fuel to complete the forward trip, and is the active satellite for the corresponding P2P maneuver. Table 2 summarizes the results for the global P2P refueling problem for the constellation under consideration. It shows the satellites, the matching pair (with whom the satellite has a fuel exchange), the initial and final fuel content in terms of the percentage of maximum fuel capacity of the satellite.

**Table 2. Summary of Results for the Example P2P Strategy.**

| Satellites | Initial Fuel | Matching Pair | Final fuel |
|---|---|---|---|
| $s_1$ | $100\%$ | $s_3$ | $64.4\%$ |
| $s_6$ | $100\%$ | $s_4$ | $35\%$ |
| $s_7$ | $100\%$ | $s_5$ | $35\%$ |
| $s_8$ | $100\%$ | $s_2$ | $35\%$ |
| $s_2$ | $1\%$ | $s_8$ | $57.2\%$ |
| $s_3$ | $7\%$ | $s_1$ | $35\%$ |
| $s_4$ | $2\%$ | $s_6$ | $58.3\%$ |
| $s_5$ | $1\%$ | $s_7$ | $57.4\%$ |

We also solve the same P2P problem for the impulsive case, that is, we assume that the satellites employ chemical propulsion system. All data (mass, fuel, and orbital position of satellites) are kept the same, except for the engine characteristic of the satellites. The specific impulse considered for the chemical propulsion system employed by the satellites is 300 sec. For the impulsive case, the total fuel expended during the refueling process is $18.6\%$ of the total initial fuel in the constellation. This is much greater than the amount expended during the low-thrust mission. In other words, this example demonstrates the benefit of a low-thrust P2P servicing mission over an impulsive P2P servicing mission in terms of the propellant expended during the refueling process.

## CONCLUSION

In this paper, we develop a tool to determine minimum fuel, time-fixed low-thrust P2P maneuvers. We formulate the optimal control problems associated with the forward and return trips of the

P2P maneuver, and provide a solution methodology for the associated two-point boundary value problems in either case. We also demonstrate the applicability of the tool in solving a complete P2P refueling scenario. It is found that similar to the impulsive case, during the P2P maneuvers, the active satellites return with just the enough amount of fuel to be considered fuel-sufficient. Also, it is observed that if a fuel-deficient satellite has enough fuel to complete the forward trip, it can become an active satellite. These observations are similar to the impulsive case. However, it is demonstrated with an example that a low-thrust P2P mission (compared to an impulsive P2P mission) means a lower percentage of propellant being used up during the refueling process. Also, this study is a first step towards realizing a distributed low-thrust servicing mission for a system of multiple satellites. In other words, we have developed a tool that will be helpful in future studies of more complex and challenging refueling strategies, for instance an Egalitarian Low-Thrust P2P scenario in which we allow the active satellites to interchange their orbital slots during their return trips.

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